THE QUANTUM ERASER

This problem deals with a quantum process where the superposition of two probability amplitudes leads to an interference phenomenon. The two amplitudes can be associated with two quantum paths, as in a double slit interference experiment. We shall first show that these interferences disappear if an intermediate measurement gives information about which path has actually been followed. Next, we shall see how interference is « erased » by a quantum device.

We consider a beam of neutrons, which are particles of charge zero and spin $\frac{1}{2}$, propagating along the x axis with velocity v. In all what follows, the motion of the neutrons in space is treated classically as a uniform linear motion. Only the evolution of their spin states is treated quantum mechanically.

1. Magnetic Resonance

The eigenstates of the z component of the neutron spin are denoted $|n:+\rangle$ and $|n:-\rangle$. A constant uniform magnetic field $\vec{B}_0 = B_0\vec{u}_z$ is applied along the z axis ($\vec{u}_z$ is the unit vector along the z axis). The magnetic moment of the neutron is denoted $\vec{\mu}_n = \gamma\,\vec{S}_n$, where γ_n is the gyromagnetic ratio and $\hat{\vec{S}}_n$ the spin operator of the neutron.

1.1. What are the magnetic energy levels of a neutron in the presence of the field $\vec{B}_0$? Express the result in term of $\omega_0 = -\gamma_n B_0$.

1.2. The neutron cross a cavity of length L between times t_0 and $t_1 = t_0 + \frac{L}{v}$. Inside this cavity, in addition to the constant field $\vec{B}_0$, a rotating field $\vec{B}_1(t)$ is applied. The field $\vec{B}_1(t)$ lies in the (x, y) plane and it has a constant angular frequency ω .

$$\vec{B}_1(t) = B_1\left(\cos\omega t\;\vec{u}_x + \sin\omega t\;\vec{u}_y\right) \qquad (1)$$

Let $|\Psi_n(t)\rangle = \alpha_+(t)|n:+\rangle + \alpha_-|n:-\rangle$ be the neutron spin state at time t , and consider a neutron entering the cavity at time t_0.

a) Write the equations of evolution for $\alpha_\pm(t)$ when $t_0 \le t \le t_1$. We set hereafter $\omega_1 = -\gamma_n B_1$.

b) Setting $\alpha_\pm(t) = \beta_\pm(t)e^{\mp i\frac{\omega(t-t_0)}{2}}$, show that the problem reduces to a differential system with constant coefficients.

c) We assume that we are near the resonance : $|\omega-\omega_0| << \omega_1$, and that terms proportional to $(\omega-\omega_0)$ may be neglected in the previous equations. Check that, within this approximation, one has, for $t_0 \le t \le t_1$,

$$\beta_\pm(t) = \beta_\pm(t_0)\cos\theta - ie^{\mp i\omega t_0}\beta_\mp(t_0)\sin\theta$$

where $\theta = \frac{\omega_1(t-t_0)}{2}$.

d) Show that the spin state at time t_1, when the neutron leaves the cavity, can be written as :

$$\begin{pmatrix} \alpha_+(t_1) \\ \alpha_-(t_1) \end{pmatrix} = \hat{U}(t_0,t_1) \begin{pmatrix} \alpha_+(t_0) \\ \alpha_-(t_0) \end{pmatrix} \quad (2)$$

where the matrix $U(t_0,t_1)$ is

$$U(t_0,t_1) = \begin{pmatrix} e^{-i\chi}\cos\phi & -ie^{-i\delta}\sin\phi \\ -ie^{i\delta}\sin\phi & e^{i\chi}\cos\phi \end{pmatrix} \quad (3)$$

and where $\phi = \dfrac{\omega_1(t_1 - t_0)}{2}$, $\chi = \dfrac{\omega(t_1 - t_0)}{2}$ and $\delta = \dfrac{\omega(t_1 + t_0)}{2}$

2. Ramsey Fringes

The neutrons are initially in the spin state $|n:-\rangle$. They successively cross two identical cavities of the type described above. This is called Ramsey configuration and it shown in Fig. 1. The same oscillating field $\vec{B}_1(t)$ given by (1), is applied in both cavities. The modulus B_1 of this field is adjusted so as to satisfy the condition $\phi = \frac{\pi}{4}$. The constant field $\vec{B}_0$ is applied throughout the experimental setup. At the end of this setup, one measure the number of outgoing neutrons which have flipped their spin and are in the final state $|n:+\rangle$. This is done for several values of ω in the vicinity of $\omega = \omega_0$.

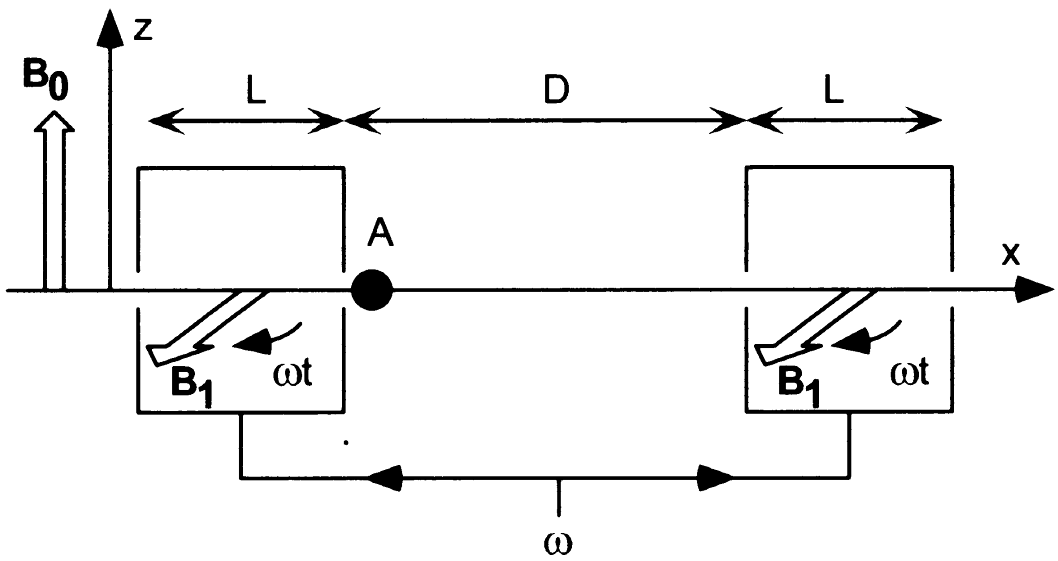

Fig. 1. : *Ramsey's configuration ; the role of the detecting atom A is specified in parts 3 and 4.*

2.1. At time t_0 a neutron enters the first cavity in the state $|n:-\rangle$. What is its spin state, and what is the probability of finding it in the state $|n:+\rangle$, when it leaves the cavity ?

2.2. The same neutron enters the second cavity at time $t'_0 = t_1 + T$ which $T = \dfrac{D}{v}$ where D is the distance between the two cavities the spin precesses freely around $\vec{B}_0$. What is the spin state of the neutron at time t'_0 ?

2.3. Let t'_1 be the time when the neutron leaves the second cavity : $t'_1 - t'_0 = t_1 - t_0$. Express the quantity $\delta' = \frac{\omega(t'_1 + t'_0)}{2}$ in terms of ω, t_0, t_1 and T. Write the transition matrix $U(t'_0, t'_1)$ in the second cavity.

2.4. Calculate the probability P_+ of detecting the neutron in the state $|n:+\rangle$ after the second cavity. Show that it is an oscillating function of $(\omega_0 - \omega)T$. Explain why this result can be interpreted as in interference process.

2.5. In practice, the velocities of the neutron have some dispersion around the mean value v. This results in a dispersion in the time T to get from one cavity to the other. A typical experimental result giving the intensity of the outgoing beam in the state $|n:+\rangle$ as a function of the frequency $\nu = \frac{\omega}{2\pi}$ of the rotating field $\vec{B}_1$ is shown in Fig.2.

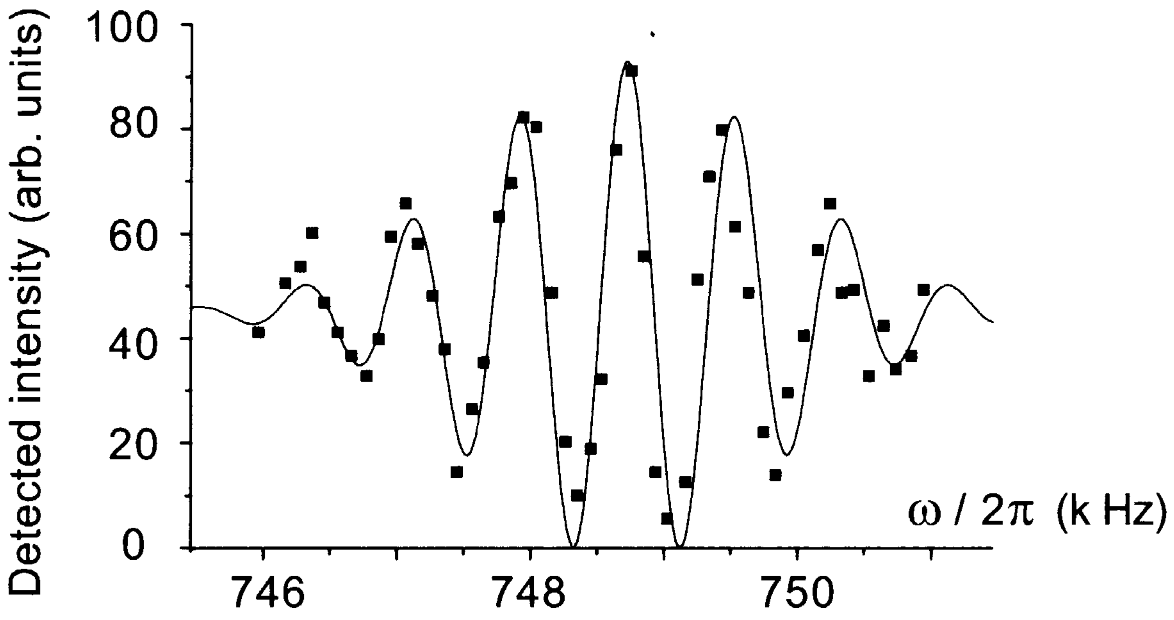

Fig.2. : *Intensity of the outgoing beam in the state $|n:+\rangle$ as a function of the frequency $\nu = \frac{\omega}{2\pi}$ for a neutron beam with some velocity dispersion.*
(J.H.Smith et al., Phys. Rev. ***108****, 120, (1957)).*

a) Explain the shape of this curve by averaging the previous result over the distribution.

$$dp(T) = \frac{1}{\tau\sqrt{2\pi}} e^{-\frac{(T-T_0)^2}{2\tau^2}} dT$$

(We recall that $\int_{-\infty}^{+\infty} \cos(\Omega T) dp(T) = e^{-\frac{\Omega^2\tau^2}{2}} \cos(\Omega T_0)$).

b) In the above experiment, the value of the magnetic field was $B_0 = 2{,}57 \times 10^{-2}\,\mathrm{T}$ and the distance $D = 1{,}6\,\mathrm{m}$. Calculate the magnetic moment of the neutron. Evaluate the average velocity $v_0 = \frac{D}{T_0}$ and the velocity dispersion $\delta v = \frac{v_0 \tau}{T_0}$ of the neutron beam.

c) Which optical interference experiment is the result reminiscent of ?

2.6. Suppose one inserts between the two cavities of Fig.1 a device which can measure the z component of the neutron spin (the principle of such detector is presented in the next section). Determine the probability $P_{+,+}$ of detecting the neutron in the state $|n:+\rangle$ between the two cavities and the probability $P_{-,+}$ of detecting the neutron in the state $|n:-\rangle$ when it leaves the second cavity. Check that one does *not* have $P_{+} = P_{+,+} + P_{-,+}$ and comment on this fact.

3. Detection of the Neutron Spin State

In order to measure the spin of a neutron, one lets it interact during a time τ with a spin $\frac{1}{2}$ atom at rest. The atom's spin operator is $\vec{S}_a$. Let $|a:\pm\rangle$ be the two eigenstate of the observable $\hat{S}_{az}$. After the interaction between the neutron and the atom, one measure the spin of the atom. Under certain conditions, as we shall see, one can deduce the spin state of the neutron after this measurement.

3.1. Spin states of the atom.

Let $|a:\pm x\rangle$ be the eigenstates of $\hat{S}_{ax}$ and $|a:\pm y\rangle$ those of $\hat{S}_{ay}$. Write $|a:\pm x\rangle$ and $|a:\pm y\rangle$ in the basis $\{|a:+\rangle, |a:-\rangle\}$. Express $|a:\pm y\rangle$ in terms of $|a:\pm x\rangle$.

3.2. We assume that the neutron-atom interaction does not affect the neutron trajectory. We represent the interaction between the neutron and the atom by a very simple model. This interaction is assumed to last a finite time τ during which the neutron-atom interaction Hamiltonian has the form

$$\hat{V} = \frac{2A}{\hbar}\hat{S}_{nz} \otimes \hat{S}_{ax} \qquad (4)$$

where A is a constant. We neglect the action of any external field, including $\vec{B}_0$, during the time τ.

Explain why $\hat{S}_{nz}$ and $\hat{V}$ commute. Give their common eigenstates and the corresponding eigenvalues.

3.3. We hereafter assume that the interaction time τ is adjusted in such a way that

$$A\tau = \frac{\pi}{2}$$

Suppose the initial state of the system is

$$|\Psi(0)\rangle = |n:+\rangle \otimes |a:+y\rangle$$

Calculate the final state of the system $|\Psi(\tau)\rangle$. Answer the same question if the initial state is

$$|\Psi(0)\rangle = |n:-\rangle \otimes |a:+y\rangle$$

3.4. We now suppose that the initial spin state is

$$|\Psi(0)\rangle = (\alpha_{+}|n:+\rangle + \alpha_{-}|n:-\rangle) \otimes |a:+y\rangle$$

After the neutron-atom interaction described above, one measure the z component $\hat{S}_{az}$ of the atom's spin.

a) What results can one find, and with what probabilities ?

b) *After* this measurement, what prediction can one make about the value of the z component of the neutron spin ? Is it necessary to let the neutron interact with another measuring apparatus in order to know $\hat{S}_{nz}$ once the value of $\hat{S}_{az}$ is known ?

4. A Quantum Eraser

We have seen above that if one measures the spin state of the atom between the two cavities, the interference signal disappears. In this section, we will show that it is possible to recover n interference if the information left by the neutron on the detecting atom is "erased" by an appropriate measurement.

A neutron, initially in the spin state $|n:-\rangle$, is sent into the two-cavity system. Immediately after the first cavity, there is a detecting atom of the type discussed above, prepared in the spin state $|a:+y\rangle$. By assumption, the spin state of the atom evolves only during the time interval τ when it interacts with the neutron.

4.1. Write the spin state of the neutron-atom system when the neutron is :

a) just leaving the first cavity (time t_1), before interacting with the atom ;

b) just after the interaction with the atom (time $t_1+\tau$) ;

c) entering the second cavity (time t'_0) ;

d) just leaving the second cavity (time t'_1) .

4.2. What is the probability of finding the neutron in the state $|n:+\rangle$ at time t'_1 ? Does this probability reflect an interference phenomenon ? Interpret the result.

4.3. At time t'_1, Bob measures the z component of the neutron spin and Alice measures the y component of the atom's spin. Assume both measurements give $+\frac{\hbar}{2}$. Show that the corresponding probability reflects an interference phenomenon.

4.4. Is This result compatible with the conclusion of question **4.2.** ?

4.5. In your opinion, which of the following three statements are appropriate, and for what reasons ?

a) When, Alice performs a measurement on the atom, Bob sees at once an interference appear in the signal he is measuring on the neutron.

b) Knowing the result obtained by Alice on each event, Bob can select a subsample of his own events which displays an interference phenomenon.

c) The experiment corresponds to an interference between two quantum paths for the neutron spin. By restoring the initial state of the atom, the measurement done by Alice erases the information concerning which quantum path is followed by the neutron spin, and allows interferences to reappear.

4.6. Alice now measures the component of the atom's spin along an arbitrary axis defined by the unit vector $\vec{w}$. Show that the contrast of the interferences varies proportionally to $|\sin\eta|$ where $\cos\eta = \vec{w}.\vec{u}_z$. Interpret the result.

Solutions

1.1. The magnetic energy levels are : $E_{\pm} = \mp\frac{\hbar}{2}\gamma_n B_0 = \pm\frac{1}{2}\hbar\omega_0$

1.2. **a)** The Hamiltonian is : $\hat{H} = \frac{\hbar}{2}\begin{pmatrix} \omega_0 & \omega_1 e^{-i\omega t} \\ \omega_1 e^{i\omega t} & -\omega_0 \end{pmatrix}$. Therefore, the evolution equations are

$$\begin{cases} i\dot{\alpha}_+ = \frac{\omega_0}{2}\alpha_+ + \frac{\omega_1}{2}e^{-i\omega t}\alpha_- \\ i\dot{\alpha}_- = -\frac{\omega_0}{2}\alpha_- + \frac{\omega_1}{2}e^{+i\omega t}\alpha_+ \end{cases}$$

b) With the variables $\beta_{\pm}(t) = \alpha_{\pm}(t)e^{\pm\frac{i\omega(t-t_0)}{2}}$, we obtain

$$\begin{cases} i\dot{\beta}_+ = -\frac{\omega-\omega_0}{2}\beta_+ + \frac{\omega_1}{2}e^{-i\omega t_0}\beta_- \\ i\dot{\beta}_- = \frac{\omega-\omega_0}{2}\beta_- + \frac{\omega_1}{2}e^{i\omega t_0}\beta_+ \end{cases}$$

c) If $|\omega-\omega_0| << \omega_1$, we have, to good approximation, the differential system :

$$\begin{cases} i\dot{\beta}_+ = \frac{\omega_1}{2}e^{-i\omega t_0}\beta_- \\ i\dot{\beta}_- = \frac{\omega_1}{2}e^{i\omega t_0}\beta_+ \end{cases}$$

whose solution is indeed

$$\beta_{\pm}(t) = \beta_{\pm}(t_0)\cos\frac{\omega_1(t-t_0)}{2} - ie^{\mp i\omega t_0}\beta_{\mp}(t_0)\sin\frac{\omega_1(t-t_0)}{2}$$

d) Defining $\phi = \frac{\omega_1(t_1-t_0)}{2}$, $\chi = \frac{\omega(t_1-t_0)}{2}$, $\delta = \frac{\omega(t_1+t_0)}{2}$ we obtain

$$\begin{cases} \alpha_+(t_1) = e^{-i\chi}\beta_+(t_1) = e^{-i\chi}\left[\alpha_+(t_0)\cos\phi - i\alpha_-(t_0)e^{-i\omega t_0}\sin\phi\right] \\ \alpha_-(t_1) = e^{i\chi}\beta_-(t_1) = e^{+i\chi}\left[\alpha_-(t_0)\cos\phi - i\alpha_+(t_0)e^{+i\omega t_0}\sin\phi\right] \end{cases}$$

and therefore

$$\hat{U} = \begin{pmatrix} e^{-i\chi}\cos\phi & -ie^{-i\delta}\sin\phi \\ -ie^{i\delta}\sin\phi & e^{i\chi}\cos\phi \end{pmatrix}$$

1.2. We assume $\phi = \frac{\pi}{4}$; the initial conditions are $\begin{cases} \alpha_+(t_0) = 0 \\ \alpha_-(t_0) = 1 \end{cases}$. At time t_1 the state is

$$|\Psi(t_1)\rangle = \frac{1}{\sqrt{2}}\left(-ie^{-i\delta}|n:+\rangle + e^{i\chi}|n:-\rangle\right)$$

In other words $\begin{cases} \alpha_+(t_1) = \dfrac{-ie^{-i\delta}}{\sqrt{2}} \\ \alpha_-(t_1) = \dfrac{e^{i\chi}}{\sqrt{2}} \end{cases}$ and $\boxed{P_\pm = \dfrac{1}{2}}$

2.2. We put $T = \dfrac{D}{v}$. The neutron spin precesses freely between the two cavities during time T, and we obtain

$$\begin{pmatrix} \alpha_+(t_0^{'}) \\ \alpha_-(t_0^{'}) \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} -ie^{-i\delta} e^{-i\frac{\omega_0 T}{2}} \\ e^{i\chi} e^{+i\frac{\omega_0 T}{2}} \end{pmatrix} \qquad (5)$$

2.3. By definition, $t_0^{'} = t_1 + T$ and $t_1^{'} = 2t_1 - t_0 + T$, therefore the transition matrix in the second cavity is

$$\hat{U}' = \begin{pmatrix} e^{-i\chi'} \cos\phi' & -ie^{-i\delta'} \sin\phi' \\ -ie^{i\delta'} \sin\phi' & e^{i\chi'} \cos\phi' \end{pmatrix}$$

with $\phi' = \phi = \dfrac{\omega_1(t_1 - t_0)}{2}$, $\chi' = \chi = \dfrac{\omega(t_1 - t_0)}{2}$. Only the parameter δ is changed into

$$\delta' = \frac{\omega(t_1^{'} + t_0^{'})}{2} = \frac{\omega(3t_1 + 2T - t_0)}{2}$$

2.4. The probability amplitude for detecting the neutron in state + after the second cavity is obtained by (i) applying the matrix $\hat{U}'$ to the vector (5), (ii) calculating the scalar product of the result with $|n:+\rangle$. We get in this way

$$\alpha_+(t_1^{'}) = \frac{1}{2}\left(-ie^{-i\left(\chi + \delta + \frac{\omega_0 T}{2}\right)} - ie^{-i\left(\delta' - \chi - \frac{\omega_0 T}{2}\right)} \right)$$

since $\delta + \chi = \omega t_1$, $\delta' - \chi = \dfrac{\omega}{2}(3t_1 + 2T - t_0 - t_1 + t_0) = \omega(t_1 + T)$

we have

$$\alpha_+(t_1^{'}) = -\frac{i}{2} e^{-i\omega\left(t_1 + \frac{T}{2}\right)} \left(e^{-i\left(\frac{\omega_0 - \omega}{2}\right)T} + e^{i\left(\frac{\omega_0 - \omega}{2}\right)T} \right) \qquad (6)$$

Therefore, the probability that the neutron spin has flipped in the two-cavity system is

$$\boxed{P_+ = \left|\alpha_+(t_1^{'})\right|^2 = \frac{1}{2}\left[1 + \cos(\omega - \omega_0)T\right] = \cos^2 \frac{(\omega - \omega_0)T}{2}}$$

With the approximation $|\omega - \omega_0| << \omega_1$, the probability for a spin flip in a single cavity is independent of ω and is equal to $\dfrac{1}{2}$. In contrast, the present result for two cavities exhibits a strong modulation of the spin flip probability, between 1 (e.g. for $\omega = \omega_0$) and 0 (e.g. for $(\omega - \omega_0)T = \pi$). This modulation results from interference process of the two quantum paths corresponding respectively to :

- a spin flip in the first cavity, and no flip in the second one,
- no flip in the first cavity and a spin flip in the second one.

Each of these paths has a probability $\frac{1}{2}$, so that the sum of the probability amplitudes (6) is fully modulated.

2.5.a. Since $\cos^2\frac{\phi}{2}=\frac{1}{2}(1+\cos\phi)$, the average probability distribution is

$$\left\langle \cos^2\frac{(\omega-\omega_0)T}{2}\right\rangle = \frac{1}{2}+\frac{1}{2}e^{-\frac{(\omega-\omega_0)^2\tau^2}{2}}\cos\left[(\omega-\omega_0)T_0\right] \qquad (7)$$

This form agree with the observed variation in ω of the experimental signal. The central maximum, which is located at $\frac{\omega}{2\pi}=748.8$ kHz correspond to $\omega=\omega_0$. For that value, a constructive interference appears whatever the neutron velocity. The lateral maxima and minima are less peaked, however, since the position of a lateral peak is velocity dependent. The first two lateral maxima correspond to $(\omega-\omega_0)T_0\simeq\pm 2\pi$. Their amplitude is reduced, compared to the central peak, by a factor $e^{-\frac{2\pi^2\tau^2}{T_0^2}}$.

b. The angular frequency ω_0 is related to the magnetic moment of the neutron by $\hbar\omega_0=2\mu_n B_0$ which leads to $\mu_n=9.65\times10^{-27}$ J.T^{-1}. The time T_0 can be deduced from the spacing between the central maximum and a lateral one. The first lateral maximum occurs at 0.77 kHz from the resonance, hence $T_0=1.3$ ms. This correspond to an average velocity $v_0=1230$ m.s^{-1}.

The ratio of intensities between the second lateral maximum and the central one is roughly 0.55. This is approximately equal to $e^{-\frac{8\pi^2\tau^2}{T_0^2}}$ and gives $\frac{\tau}{T_0}\approx 0.087$, and $\delta v\approx 110$ m.s^{-1}.

c. This experiment can be compared to a Young's double slit interference experiment with polychromatic light. The central fringe (corresponding to the peak at $\omega=\omega_0$) remains bright, but the contrast of the interferences decreases rapidly as one departs from the center. In fact, the maxima for some frequencies correspond to minima for others.

2.6. The probability P_{++} is the product of the two probabilities : the probability of finding the neutron in the state $|n:+\rangle$ when it leaves the first cavity $\left(p=\frac{1}{2}\right)$ and, knowing that it is in the state $|n:+\rangle$, the probability of finding it in the same state when it leaves the second cavity $\left(p=\frac{1}{2}\right)$; this gives $P_{++}=\frac{1}{4}$. Similarly $P_{-+}=\frac{1}{4}$. The sum $P_{++}+P_{-+}=\frac{1}{2}$ does not display any interference, since one has measured in which cavity the neutron spin has flipped. This is very similar to an electron double-slit interference experiment if one measures through which slit the electron passes.

3.1. By definition :

$$|a:\pm x\rangle=\frac{1}{\sqrt{2}}\left(|a:+\rangle\pm|a:-\rangle\right)$$

$$|a:\pm y\rangle=\frac{1}{\sqrt{2}}\left(|a:+\rangle\pm i|a:-\rangle\right)$$

and these states are related to one another by

$$|a:\pm y\rangle = \frac{1}{2}\left[(1\pm i)|a:+x\rangle + (1\mp i)|a:-x\rangle\right]$$

3.2. The operators S_{nz} and S_{ax} commute since they act in two different Hilbert spaces ; therefore $\left[\hat{S}_{nz}, \hat{V}\right] = 0$.

The common eigenvectors of $\hat{S}_{nz}$ and $\hat{V}$ and the corresponding eigenvalues are :

$$|n:+\rangle \otimes |a:\pm x\rangle \qquad S_{nz} = +\frac{\hbar}{2} \qquad V = \pm\frac{A\hbar}{2}$$

$$|n:-\rangle \otimes |a:\pm x\rangle \qquad S_{nz} = -\frac{\hbar}{2} \qquad V = \mp\frac{A\hbar}{2}$$

The operators $\hat{S}_{nz}$ and $\hat{V}$ form a complete set of commuting operators as far as spin variables are concerned.

3.3. Expanding in terms of the energy eigenstates, one obtains for $|\Psi(0)\rangle = |n:+\rangle \otimes |a:+y\rangle$:

$$|\Psi(\tau)\rangle = \frac{1}{2}|n:+\rangle \otimes \left[(1+i)e^{-i\frac{A\tau}{2}}|a:+x\rangle + (1-i)e^{i\frac{A\tau}{2}}|a:-x\rangle\right]$$

that is to say, for $\dfrac{A\tau}{2} = \dfrac{\pi}{4}$:

$$|\Psi(\tau)\rangle = \frac{1}{\sqrt{2}}|n:+\rangle \otimes \left(|a:+x\rangle + |a:-x\rangle\right) = |n:+\rangle \otimes |a:+\rangle$$

Similarly, if $|\Psi(0)\rangle = |n:-\rangle \otimes |a:+y\rangle$, then $|\Psi(\tau)\rangle = i|n:-\rangle \otimes |a:-\rangle$.

Physically, this means that the neutron's spin state does not change since it is an eigenstate of $\hat{V}$, while the atom's spin precesses around the x axis with angular frequency A. At time $\tau = \dfrac{\pi}{2A}$, it lies along the z axis.

3.4. If the initial state is $|\Psi(0)\rangle = \left[\alpha_+|n:+\rangle + \alpha_-|n:-\rangle\right] \otimes |a:+y\rangle$, the state after the interaction is

$$|\Psi(\tau)\rangle = \alpha_+|n:+\rangle \otimes |a:+\rangle + i\alpha_-|n:-\rangle \otimes |a:-\rangle$$

The measurement of the z component of the atom's spin gives $+\dfrac{\hbar}{2}$, with probability $|\alpha_+|^2$ and state $|n:+\rangle \otimes |a:+\rangle$ after the measurement, or $-\dfrac{\hbar}{2}$ with probability $|\alpha_-|^2$ and state $|n:-\rangle \otimes |a:-\rangle$ after the measurement.

In both cases, after measuring the z component of the atom's spin, the neutron spin state is known : it is the same as that of the measured atom. It is *not* necessary to let the neutron interact with another measuring apparatus in order to know the value of S_{nz}.

4.1. The successive states are :

step (a) $\quad \frac{1}{\sqrt{2}}\left(-ie^{-i\delta}|n{:}+\rangle\otimes|a{:}+y\rangle+e^{i\chi}|n{:}-\rangle\otimes|a{:}+y\rangle\right)$

step (b) $\quad \frac{1}{\sqrt{2}}\left(-ie^{-i\delta}|n{:}+\rangle\otimes|a{:}+\rangle+ie^{i\chi}|n{:}-\rangle\otimes|a{:}-\rangle\right)$

step (c) $\quad \frac{1}{\sqrt{2}}\left(-ie^{-i\left(\delta+\frac{\omega_0 T}{2}\right)}|n{:}+\rangle\otimes|a{:}+\rangle+ie^{i\left(\chi+\frac{\omega_0 T}{2}\right)}|n{:}-\rangle\otimes|a{:}-\rangle\right)$

Finally, when the neutron leaves the second cavity (step d), the state of the system is :

$$|\Psi_f\rangle=\frac{1}{2}\left[-ie^{-i\left(\delta+\frac{\omega_0 T}{2}\right)}\left(e^{-i\chi}|n{:}+\rangle-ie^{i\delta'}|n{:}-\rangle\right)\otimes|a{:}+\rangle+ie^{i\left(\chi+\frac{\omega_0 T}{2}\right)}\left(-ie^{i\delta'}|n{:}+\rangle+e^{i\chi}|n{:}-\rangle\otimes|a{:}-\rangle\right)\right]$$

4.2. The probability of finding the neutron in state $|+\rangle$ is the sum of the probabilities for finding :

- the neutron in state + and the atom in state +, i.e. the square of the modulus of the coefficient of $|n{:}+\rangle\otimes|a{:}+\rangle$ $\left(\frac{1}{4}\text{ in the present case}\right)$,
- the neutron in state + and the atom in state - (probability $\frac{1}{4}$ again).

One gets therefore $P_+=\frac{1}{4}+\frac{1}{4}=\frac{1}{2}$: There are no interferences since the quantum path leading in the end to a spin flip of the neutron can be determined from the state of the atom.

4.3. One can expand the vectors $|a{:}\pm\rangle$ on $|a{:}\pm y\rangle$:

$$|\Psi_f\rangle=\frac{1}{2\sqrt{2}}\begin{bmatrix}-ie^{-i\left(\delta+\frac{\omega_0 T}{2}\right)}\left(e^{-i\chi}|n{:}+\rangle-ie^{i\delta'}|n{:}-\rangle\right)\otimes\left(|a{:}+y\rangle+|a{:}-y\rangle\right)\\ +e^{i\left(\chi+\frac{\omega_0 T}{2}\right)}\left(-ie^{-i\delta'}|n{:}+\rangle+e^{i\chi}|n{:}-\rangle\otimes\left(|a{:}+y\rangle-|a{:}-y\rangle\right)\right)\end{bmatrix}$$

The probability amplitude that Bob finds $+\frac{\hbar}{2}$ along the z axis while Alice finds $+\frac{\hbar}{2}$ along the y axis is the coefficient of the term $|n{:}+\rangle\otimes|a{:}+y\rangle$ in the above expansion. Equivalently, the probability is obtained by projecting the state onto $|n{:}+\rangle\otimes|a{:}+y\rangle$, and squaring. One obtains

$$P\left(S_{nz}=\frac{\hbar}{2},S_{ay}=\frac{\hbar}{2}\right)=\frac{1}{8}\left|-ie^{-i\left(\delta+\chi+\frac{\omega_0 T}{2}\right)}-ie^{i(\chi-\delta')+\frac{\omega_0 T}{2}}\right|^2=\frac{1}{2}\cos^2\frac{(\omega-\omega_0)T}{2}$$

which clearly exhibits a modulation reflecting an interference phenomenon. Similarly, one finds that

$$P\left(S_{nz}=\frac{\hbar}{2},S_{ay}=-\frac{\hbar}{2}\right)=\frac{1}{2}\sin^2\frac{(\omega-\omega_0)T}{2}$$

which is also modulated.

4.4. This result is compatible with the result **4.2.** Indeed the sum of the two probabilities above is $\frac{1}{2}$ as in **4.2.** If Bob does not know the result found by Alice, or if Alice does not perform a measurement, which is equivalent from his point of view, Bob sees no interferences. The interferences only arise for the joint probability $P\left(S_{nz},S_{ay}\right)$.

4.5.

a) This first statement is obviously wrong. As seen in question **4.2.**, if the atom A is present, Bob no longer sees oscillations $(\text{in } \omega-\omega_0)$ of the probability for detecting the neutron in the state $|+\rangle$. This probability is equal to $\frac{1}{2}$ whatever Alice does. Notice that if the statement were correct, this would imply instantaneous transmission of information from Alice to Bob. By seeing interferences appear, Bob would know immediately that Alice is performing an experiment, even though she may be very far away.

b) This second statement is correct. If Alice and Bob put together all their results, and if they select the subsample of events for which Alice finds $+\frac{\hbar}{2}$, then the number of events for which Bob also finds $+\frac{\hbar}{2}$ varies like $\cos^2\left(\frac{(\omega-\omega_0)T}{2}\right)$; they recover interferences for this subset of events. In the complementary set, where Alice has found $-\frac{\hbar}{2}$, the number of Bob's results giving $+\frac{\hbar}{2}$ varies like $\sin^2\left(\frac{(\omega-\omega_0)T}{2}\right)$. This search for correlations between events occurring in different detectors is a common procedure, in particular physics for instance.

c) This third statement, although less precise but more picturesque than the previous one, is nevertheless acceptable. The $\cos^2\left(\frac{(\omega-\omega_0)T}{2}\right)$ signal found in **1.2.** can be interpreted as the interference of the amplitudes corresponding to two quantum paths for the neutron spin which is initially in the state $|n{:}-\rangle$; either its spin flips in the first cavity, or it flips in the second one. If there exists a possibility to determine which quantum path is followed by the system, interferences cannot appear. It is necessary to « erase » this information, which is carried by the atom, in order to observe « some » interferences. after Alice has measured the atom's spin along the y axis, she has, in some sense « restored » the initial state of the system, and this enables Bob to see some interferences. It is questionable to say that information has been erased : one may feel that, on the contrary, extra information has been acquired. Notice that the statement in the text does not specify in which physical quantity the interferences appear. Notice also that the order of the measurements made by Alice and Bob has no importance, contrary to what this third statement seems to imply.

4.6. Alice can measure along the axis $\vec{w}=\sin\eta\,\vec{u}_y+\cos\eta\,\vec{u}_z$, in the (y,z) plane, for instance. Projecting $|\Psi_f\rangle$ onto the eigenstate of S_{aw} with eigenvalue $+\frac{\hbar}{2}$, i.e.

$$\cos\left(\frac{\eta}{2}\right)|a{:}+\rangle+i\sin\left(\frac{\eta}{2}\right)|a{:}-\rangle$$

a calculation similar to **4.3.** leads to a probability $\frac{1}{2}\left[1+\sin\eta\cos\left((\omega-\omega_0)T\right)\right]$. If $\eta=0$ or π (measurement along the z axis) there are no interferences. For $\eta=\frac{\pi}{2}$ and $\frac{3\pi}{2}$ or, more generally, if Alice measures in the (x,y) plane, the contrast of the interferences, $|\sin\eta|$, is maximum.